9-1 day 5 The Ratio and Root Tests

Learning Objectives:

I can use the ratio test to determine whether an infinite series converges or diverges

I can use the root test to determine whether an infinite series converges or diverges

Jan 31-1:58 PM

The Ratio Test

Let $\sum a_n$ be a series with non-zero terms

1.) $\sum a_n$ converges absolutely if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|<1$

2.) $\sum a_n$ diverges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|>1$

3.) The ratio test in inconclusive if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|=1$

Jan 31-1:58 PM

Ex1. Use the Ratio Test to determine if each series converges.

1.) $\sum_{n=0}^{\infty}\frac{2^n}{n!}$ $a_n = \frac{2^n}{n!}$ $a_{n+1} = \frac{2^{n+1}}{(n+1)!}$

$\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{\frac{2^{n+1}}{(n+1)!}}{\frac{2^n}{n!}}\right|$

$= \lim_{n\to\infty}\frac{2^{n+1}}{(n+1)!}\cdot\frac{n!}{2^n} = \lim_{n\to\infty}\frac{2^n\cdot 2^1}{(n+1)\cdot n!}\cdot\frac{n!}{2^n}$

$= \lim_{n\to\infty}\frac{2}{n+1} = 0$ converges

Jan 31-1:58 PM

2.) $\sum_{n=1}^{\infty}\frac{n^2\cdot 2^{n+1}}{3^n}$

$\lim_{n\to\infty}\left|\frac{\frac{(n+1)^2\cdot 2^{n+2}}{3^{n+1}}}{\frac{n^2\cdot 2^{n+1}}{3^n}}\right| = \lim_{n\to\infty}\frac{(n+1)^2\cdot 2^{n+2}}{3^{n+1}}\cdot\frac{3^n}{n^2\cdot 2^{n+1}}$

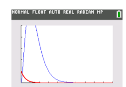
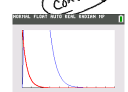
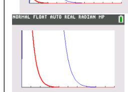
$= \lim_{n\to\infty}\frac{(n+1)^2\cdot 2^n\cdot 2^2}{3^n\cdot 3^1}\cdot\frac{3^n}{n^2\cdot 2^n\cdot 2^1}$

$= \lim_{n\to\infty}\frac{2(n+1)^2}{3n^2} = \lim_{n\to\infty}\frac{2(n^2+2n+1)}{3n^2}$

$= \lim_{n\to\infty}\frac{2n^2+4n+2}{3n^2}$ $\frac{\infty}{\infty}$ L'H R

$= \lim_{n\to\infty}\frac{4n+4}{6n}$ $\frac{\infty}{\infty}$

$= \lim_{n\to\infty}\frac{4}{6} = \frac{2}{3} < 1$ converge

Jan 31-1:58 PM

3.) $\sum_{n=1}^{\infty}\frac{n^n}{n!}$

$= \lim_{n\to\infty}\frac{(n+1)^{n+1}}{(n+1)!}\cdot\frac{n!}{n^n}$

$= \lim_{n\to\infty}\frac{(n+1)^n\cdot(n+1)}{(n+1)\cdot n!}\cdot\frac{n!}{n^n} = \lim_{n\to\infty}\frac{(n+1)^n}{n^n}$

$= \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ 1^∞

$y = \left(1+\frac{1}{n}\right)^n$

$\ln y = \ln\left(1+\frac{1}{n}\right)^n$

$= n\cdot\ln\left(1+\frac{1}{n}\right)$

$= \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}}$

$\lim_{n\to\infty}\frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}} = \frac{0}{0}$ L'H R

$\lim_{n\to\infty}\frac{\frac{1}{1+\frac{1}{n}}\cdot\frac{-1}{n^2}}{\frac{-1}{n^2}}$

$\lim_{n\to\infty}\ln y = 1$ $\lim_{n\to\infty}\frac{1}{1+\frac{1}{n}} = 1$

$e^1 = e > 1$

diverges

Jan 31-1:58 PM

4.) $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{n+1}$

$\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+1}\cdot\sqrt{n+1}}{n+2}}{\frac{(-1)^n\sqrt{n}}{n+1}}\right| = \lim_{n\to\infty}\frac{\sqrt{n+1}}{n+2}\cdot\frac{n+1}{\sqrt{n}}$

$= \lim_{n\to\infty}\frac{n+1}{n+2}\cdot\frac{\sqrt{n+1}}{\sqrt{n}}$

$= \lim_{n\to\infty}\frac{n+1}{n+2}\cdot\sqrt{\frac{n+1}{n}}$

$\lim_{n\to\infty}\frac{n+1}{n+2}\sqrt{1+\frac{1}{n}}$

$(1\cdot 1) = 1$

Jan 31-1:58 PM

$\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{n+1} = -\frac{1}{2}+\frac{\sqrt{2}}{3}-\frac{\sqrt{3}}{4}+\frac{2}{5}+\dots$

alt Series Test

$\lim_{n\to\infty}\frac{\sqrt{n}}{n+1}$ $\frac{\infty}{\infty}$ L'HR

$\lim_{n\to\infty}\frac{n^{1/2}}{n+1} = \lim \frac{1/2\, n^{-1/2}}{1} = \lim \frac{\frac{1}{2\sqrt{n}}}{1} = 0$

converge

Abs? or Cond?

$\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n+1}$ L.C.T.

$\frac{\sqrt{n}}{n+1} \Rightarrow \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}} = \frac{1}{n^{1/2}}$ diverge $p<1$ p-test

$\lim_{n\to\infty}\frac{\frac{\sqrt{n}}{n+1}}{\frac{1}{\sqrt{n}}} = \lim_{n\to\infty}\frac{\sqrt{n}}{n+1}\cdot\frac{\sqrt{n}}{1} = \lim_{n\to\infty}\frac{n}{n+1} = 1$

comparable

both diverge

converge conditionally

Feb 23-12:25 PM

The Root Test

Let $\sum a_n$ be a series

1.) $\sum a_n$ converges absolutely if $\lim_{n\to\infty}\sqrt[n]{|a_n|} < 1$

2.) $\sum a_n$ diverges if $\lim_{n\to\infty}\sqrt[n]{|a_n|} > 1$

3.) The root test in inconclusive if

$$\lim_{n\to\infty}\sqrt[n]{|a_n|} = 1$$

Jan 31-1:58 PM

Ex2. Use the Root Test to determine if the series converges.

$\sum_{n=1}^{\infty}\frac{e^{2n}}{n^n} = \sum_{n=1}^{\infty}\left(\frac{e^2}{n}\right)^n$

$\lim_{n\to\infty}\sqrt[n]{\left(\frac{e^2}{n}\right)^n} = \lim_{n\to\infty}\frac{e^2}{n} = 0$

$0<1$

Converges

Jan 31-1:58 PM

Homework

Ra�o and Root Test worksheet

Jan 31-1:58 PM